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Mr. Jeff Pohle
Division of Waste Management
Mail Stop 623-SS
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555

Dear Jeff:

I am enclosing a copy of Moench's paper entitled "Double-Porosity Models for a Fissured Groundwater Reservoir with Fracture Skin" that appeared in Water Resources Research in 1984. I also have enclosed the Comments associated with that article by Moench and by myself that appeared in Water Resources Research in 1985. Please call if you have any questions.

Sincerely,

Roy E. Williams
Roy E. Williams

REW:s1

enclosure

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Comment on "Double-Porosity Models for a Fissured Groundwater Reservoir With Fracture Skin" by Allen F. Moench

ROY E. WILLIAMS

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This paper constitutes a commendable effort to summarize and expand the theoretical basis for the analysis of drawdown data for fractured terrane that can be characterized by matrix blocks. The method applies porous media theory to fractured rocks by defining saturated hydraulic conductivity of the fracture system as the product of the hydraulic conductivity by the ratio of the total volume of the openings to the bulk volume of the rock (block or fracture) according to the method described by Gringarten [1982]. Moench [1984] incorporated the effect of a mineralized film (skin) on the wall of the fractures where the skin has its own separate hydraulic conductivity and its own thickness. The geometries of the block are considered in the controlling equation for the fissured network via the control of the geometry on permeability distribution and on boundary conditions. In several respects this approach is similar to the standard approach for leaky aquifers when leakage is derived from storage in the confining layer, except for the addition of the hydraulic properties and thickness of the mineralized layer on the walls of the fractures. By varying the values of the hydraulic properties of the fracture system and the mineralized layer on the walls of the fractures, virtually any drawdown curve can be simulated by the theory. Not surprisingly the drawdown curves produced by the method (type curves) are similar in shape to standard leaky aquifer curves within the influence of boundaries or with the influence of partial penetration. The method could in fact produce curves identical to those of Hantush [1960] [see Lohman, 1979] given appropriate geometry and the absence of a skin on the fractures.

This observation brings me to the major purpose of this critique. That purpose is to attempt to elucidate the plight (or perhaps the responsibility) of the field hydrogeologist under the current state of the art status of aquifer testing technology. I refer to the condition of the field hydrogeologist as a plight because he must conceptualize the hydrogeologic environment in some manner prior to deciding which theoretical analysis is appropriate to the particular hydrogeologic conditions with which he must deal at a specific site. I have alluded to this problem above by noting the similarity between the curves produced by this method and leaky drawdown curves affected by partial penetration and/or barrier boundaries. Moench has applied a solution to the aforementioned controlling equations to drawdown data from two wells at the Nevada Test Site. Well UE-25b #1 is the pumping well and well UE-25a #1 is the observation well. The wells are only 107 m apart. Moench points out correctly that five major zones of water entry over a depth interval of about 400 m occur in the pumping well. These data were obtained from a borehole flow survey log of the pumping well. The borehole flow survey log was obtained using the trace ejector method while the pump-

ing well was being pumped. Moench used the results of the third of three pumping tests as field drawdown data for matching purposes. Because the pumping test was conducted over the entire producing interval, he necessarily treated all the producing zones in the pumping well as one homogeneous aquifer even though the discrete producing zones cover a vertical section 400 m thick. This observation is pertinent for two reasons. Reason one is that partial penetration effects may have been operative because the distance between the two wells is only about one fourth the thickness of the total assumed aquifer and because it is impossible to ascertain that all the producing zones in the pumping well were intercepted by the observation well. It is almost certain that the producing zones at this site are not horizontal. The second reason is that the borehole flow survey log shows that the third and fourth producing zones in the pumping well are separated by about 200 m of very tight rock. The other producing zones are separated by between 10 and 50 m of very tight rock. Consequently, in terms of standard aquifer terminology, the borehole flow survey data show that the producing zones (whatever geologic feature they may be caused by) could be interpreted as a multiple aquifer system wherein the producing zones are separated by rock with very low saturated hydraulic conductivity. Data are not yet available on the hydraulic properties of the individual producing zones (aquifers?). The importance of this second reason is that the collective behavior of the individual producing zones would produce a drawdown curve that reflects both boundaries and leakage, depending on the characteristics of the individual producing zones penetrated by each well. It is important to note also that the available reports do not show that a borehole flow survey has been run on the observation well. Consequently, in the absence of knowledge about the producing zones in the observation well, Moench was forced to assume that the aquifer(s) in the observation well is homogeneous and identical to the aquifer(s) in the pumping well. He had no choice but to make that assumption. In this regard it is important to note that the drawdown at the end of test 3 in the pumping well was about 10.41 m, whereas the drawdown in the observation well was only 0.64 m. The pumping test lasted approximately 3 days at a pumping rate of 35.8 L/s (567.5 gal/min). Because the wells are only 107 m apart, this small drawdown in the observation well may suggest that only a portion of the producing zones (aquifers?) identified in the borehole flow survey of the pumping well responded in the observation well. Admittedly, other interpretations of these data are defensible also. Furthermore, according to the data file for the two wells and Lohmeyer *et al.* [1983], the observation well did not respond at all to pumping test 1. No discussion is presented in the data base about why this was the case. This test was conducted for 4 days at a pumping rate of 14 L/s (222 gal/min). These observations illustrate that the effects of partial penetration may have been operative at the observation well because the observation well did not penetrate all the producing zones penetrated by the

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pumping well, all of which are separated by tight rock according to the borehole flow survey. A similar situation at an NRC licensed site has been interpreted to mean that the observation well is simply on the margin of the fault controlled aquifer in which the pumping well is located [see *White and Gainer, 1984*].

Moench decided to analyze the drawdown data by assuming slab-shaped blocks because of "the scale of the problem and the observation that the distances between the two wells and the average distance between the zones of water entry shown in Figure 9 are of the same magnitude (about 100 meters)." Moench points out that he believes more closely spaced water entries would be needed to justify the use of sphere shaped blocks. I suggest that other choices are defensible as well. Moench's choice necessarily forces the assumption that the blocks, in fact, exist, that the mineralized coating on the walls of the fractures is actually reflected in the pump test data, that leakage in the usual sense of the word is not occurring, and that all the water producing zones in the pumping well are penetrated by and reflected in the drawdown in the observation well, in spite of the high pumping rate and the low drawdown in the observation well. In this particular case, vertical and horizontal hydraulic continuity and hydraulic gradient between all the producing zones in the pumping well and in the observation well are questionable and very important. Gradient is important because it appears that flow at this location is predominantly horizontal (no change in head with depth). One could erroneously interpret this to mean that vertical hydraulic continuity exists at the site (discussed below). It might be possible to demonstrate such continuity by packing off individual producing zones in the pumping well and in the observation well and investigating responses to pumping them separately.

Moench brings up the problem of the effects of partial penetration in his paper. However, he states that it probably is not important in this well test because the major zone of production appears to have been fully penetrated by the pumping well. He states also that there is evidence that there is good hydraulic connection between producing zones (in the pumping well). As I suggested above, he probably is correct that the pumping well fully penetrates all the producing zones, but it is not at all clear that the observation well penetrates all the same producing zones because the geometry of the producing zones cannot be interpreted. I point out above that the only field evidence for good vertical hydraulic connection between the producing zones in the pumping well consists of the fact that heads measured in each of the producing zones in the pumping well after isolation by packers are nearly identical. I repeat that this observation does not mean necessarily that there is good vertical hydraulic connection between the zones. Individual aquifers can have identical hydraulic heads with virtually no hydraulic connection between them. The only requirement is that equipotential lines in the producing zones be vertical. Core permeability data from the pumping well and slug tests in the pumping well suggest that the nonproducing zones are much less permeable than the producing zones. In addition, under steady state conditions, which Yucca Mountain presumably has reached since the Pleistocene, the boundaries on a flow system determine to a large extent the hydraulic head distribution within that flow system. Moench also points out that the effects of anisotropy probably are significant. He points out correctly that a well test with data from a pumped well and a single observation well is insufficient to evaluate the anisotropy.

Lastly, he points out that it is possible also that hydraulic

boundaries due to major faults or intrusive dikes and sills are present within the flow regime. He points out correctly that the change in slope that occurs at 1000 min on the drawdown curve might be interpreted by taking these factors into account. He states, however, that the change in slope is on the order of 10 to 1 rather than 2 to 1, the latter of which is characteristic of a single hydraulic boundary. This slope could easily be affected by the multiple aquifer system in combination with one or more boundaries.

The last sentence in Moench's article (prior to conclusion) merits thought. He states, "Also, as the data appear to be consistent with the assumptions of the double porosity model it is not necessary to call upon added complications." This statement merits thought because it reflects to a large extent the plight of the field hydrogeologist. If these "complications" actually exist and are not recognized in the field, then the hydraulic property values derived for the fracture system, the blocks, and the mineral coatings on the walls of the fractures will apply to some other conceptual model. They may reflect vertical saturated hydraulic conductivity values of confining layers, they may reflect multiple layered leaky aquifers, they may reflect barrier boundaries or recharge boundaries, or they may simply reflect the effect of partially penetrating wells. These questions should be resolved by some type of independent field evidence about the hydrogeologic framework for the system. Unfortunately, available technology in our profession restricts to a considerable extent the feasibility of this approach. Research is needed badly in this area.

One method of approaching the problem of obtaining reliable field data to characterize fractured aquifer systems is to study the distribution of permeability from inside the aquifer. We have attempted to accomplish this objective by gaining access to fractured aquifer systems via hard rock mines [see *Williams, 1982, and Riley et al., 1984*]. Preliminary interpretations of water production characteristics in drifts and drill holes in the vicinity of the hard rock mines that we have evaluated suggest that the major producing zones are fault controlled rather than controlled by discrete fractures. Major faults in particular drain over a long period of time, whereas fractures generally drain quickly as a drift proceeds. Inclined or horizontal drill holes that intersect faults soon tend to discharge at a relatively steady state, whereas drill holes that do not intersect faults tend to approach zero flow soon after completion. Perhaps eventually research conducted inside fractured aquifer systems will reduce the number of alternative interpretations of drawdown data observed during pumping tests and ultimately improve the "plight" of the field hydrogeologist who must fit the hydrogeologic regime to a conceptual model so that it can be tested properly.

Finally, I reemphasize the fact that my purpose here is not to discount the importance of Moench's paper and his work. I have little doubt about the fact that his interpretation and analysis of the drawdown data from wells UE-25a #1 and UE-25b #1 can be defended, except possibly for the effects of the observation well not penetrating all of the same producing zones as the pumping well. My purpose is to emphasize that on the basis of the existing data base the solution is not unique. Other conceptual models certainly can be defended also. Subsequent analyses such as travel time calculations must depend heavily on selecting the correct conceptual model. Therein lies the problem.

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Reply

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I thank Williams [this issue] for taking the time to comment on my paper [Moench, 1984] and providing me with the opportunity to respond to his thoughts regarding my paper and the "plight" of the field hydrogeologist. The primary purpose of my paper was to provide a physical mechanism, backed by a mathematical model, for resolving a conflict between two theories for flow to a well in a double-porosity formation. One theory, proposed by Warren and Root [1963], makes the assumption that fluid flow from blocks to fissures occurs under conditions referred to as "pseudo-steady state." The other theory, proposed by Kazemi [1969], assumes fully transient block-to-fissure flow. The two theories each appear to be supported by well-test data in the disciplines of groundwater hydrology and petroleum engineering. In my paper I show that the two theories can be unified by incorporating fracture skin in the mathematical model. I surmise that Williams has no disagreement with the theoretical part of my paper.

The major problem Williams has with my paper appears to stem from his concern for the field hydrogeologist, who being confronted with a multitude of type curves, must choose one that is suited to the hydrogeological conditions at a specific field site. Williams points out the "similarity between the curves produced by [the proposed] method and leaky drawdown curves affected by partial penetration and/or barrier boundaries" and says that with the introduction of fracture skin "virtually any drawdown curve can be simulated by the method." Although I agree that the method is flexible and permits the hydrogeologist to generate a large number of type curves, it is hardly capable of explaining all drawdown data for flow to a pumped well. With regard to the problem of dealing with numerous type curves, I submit that it is the responsibility of the hydrogeologist to do exactly as Williams suggests; namely, "conceptualize the hydrogeologic environment in some manner prior to deciding which theoretical analysis is appropriate ... at a specific site." All available geologic, geophysical, and hydrologic data must be considered in order to narrow down the number of alternative interpretations. Properly designed well tests then allow the hydrogeologist to confirm or reject a given conceptualization. It is risky to attempt such a conceptualization on the basis of well-test data alone.

Williams takes issue with my interpretation of the well-test data (test 3) from wells UE-25b#1 and UE-25a#1 at the Nevada Test Site. He implies that the data do not necessarily support the dual-porosity model for a fissured aquifer with fracture skin and can be explained equally well, or perhaps preferably, by classical methods involving leaky aquifers, partial penetration, and barrier boundaries. I agree that alternative explanations for these data can be found. I have provided one interpretation that I believe explains the observed hy-

drologic and geologic data. It is a simple model of a rather complex system.

Williams objects to my treatment of the 400-m-thick production zone, as evidenced by the borehole flow survey, as a single aquifer. He says that because the borehole flow survey shows several zones of water entry separated by tight rock of varying thicknesses (ranging from 10 to 200 m) the entire production zone could be interpreted as a multiple-aquifer system. He states that this is important because "the collective behavior of the individual producing zones would produce a drawdown curve that reflects both boundaries and leakage, depending on the characteristics of the individual producing zones." Williams also states that because of the proximity (110 m) of the observation well to the pumped well, effects of partial penetration may have been operative. He notes that no borehole flow survey was obtained for the observation well and that I was therefore forced to assume that the aquifer in the vicinity of the observation well is identical to the aquifer in the vicinity of the pumped well. Also he notes the "small" drawdown in the observation well at the end of test 3 and the fact that "according to the data file for the two wells and Lobmeyer *et al.* (1983), the observation well did not respond at all to pumping test 1."

The suggestion that effects of partial penetration are operative addresses the question of whether or not the system under study is homogeneous in its hydraulic properties. If an aquifer can be assumed to be homogeneous (as required by double-porosity models) and the pumped well is fully penetrating, there will clearly be no partial penetration effects observed in a partially penetrating observation well regardless of its proximity to the pumped well. If it can be convincingly demonstrated that the assumption of aquifer homogeneity is invalid for the test in question then Williams is correct that effects of partial penetration will be important. Under these circumstances the proposed double-porosity model will not apply. For the scale of this test, however, it appears that treating the 400-m-thick zone of production as a single aquifer and assuming that the aquifer in the vicinity of the observation well is the same as the aquifer around the pumped well are good approximations to reality. This conclusion is partially supported by the fact that the observation well, which is located only 110 m from the pumped well, penetrates the same formations at about the same depths as the pumped well. At a depth of 762 m the observation well penetrates about two thirds the thickness of the production zone. It is not necessary, however, that the same producing zones in the pumped well be intersected by the observation well. It is necessary only that they be interconnected, a fact that appears to be clearly demonstrated, as is explained below.

As Williams points out, it is important to establish whether or not there is vertical hydraulic continuity between all the producing zones. The evidence in support of hydraulic continuity is as follows. (1) The head in each of the producing zones after isolation by packers is nearly identical. This condition would be highly fortuitous if there were poor or no hydraulic connection between producing zones. This is especially true in

light of the real possibility that water from occasional intense precipitation events on Yucca Mountain may reach the saturated zone in spite of its great depth. (2) Examination of acoustic televiewer logs in the pumped well and cores from both the pumped well and observation well reveals the existence of numerous steeply dipping fractures and faults. Staining on the fracture surfaces suggests that many are water bearing. The absence of major zones of water entry in the middle section of the aquifer may be due, in part, to the tendency of the drill holes to deviate from the vertical in a direction parallel to the dip of the fault planes. Also, the probability of the well intersecting near-vertical fractures is small. (3) With regard to the charge that the observation well is not responding as though it were fully penetrating, the "small" drawdown in the observation well at the end of the test is what it should be based upon a simple distance-drawdown calculation for a homogeneous aquifer. This calculation makes use of the independent determination of the product of hydraulic conductivity and reservoir thickness (KH) from the late time data for the pumped well [Moench, 1984, Figure 14b]. Also, the absence of measured drawdown in the observation well for test 1 reported by Lobmeyer *et al.* [1983] is due to the fact that no pressure measurements were made in the observation well during test 1 (D. H. Lobmeyer, oral communication, 1984). It is probable that drawdown did in fact occur in the observation well during test 1. (4) The most convincing evidence for horizontal and vertical continuity of the fluid in the fracture network around wells UE-25b #1 and UE-25a #1 comes from the analysis of a tracer test [Waddell, 1984]. Waddell packed off the lowermost producing horizon in well UE-25b #1 (located between 866 and 872 m below land surface) and placed dissolved sodium bromide in well UE-25a #1. Breakthrough of sodium bromide occurred 2 days after the onset of pumping from the packed-off zone.

There are several reasons for my rejection of Williams' suggestion that the entire production zone could be interpreted as a multiple-aquifer system. I agree that in the absence of fracture skin, dual-porosity models may produce type curves that are similar, if not identical, too type curves for multiple-aquifer systems. That this is true has been pointed out by numerous investigators [e.g., Gringarten, 1982; Streltsova, 1982]. However, even if the drawdown data for wells UE-25b #1 and UE-25a #1 were to fit such type curves, which they do not, it would not be correct to describe a clearly fractured network as a multiple-aquifer system because of the contrary geological information. Also, the 10-to-1 change in slope indicated at 1000 min in Figure 4b [Moench, 1984] would, if due to barrier boundaries, require a rather complicated configuration of boundaries all located at about the same distance from the pumped well. Such a system would very likely have caused a similar change in slope in the observation-well data. No break in slope was observed [Moench, 1984, Figure 4a]. Another compelling reason for my rejection of the multiple-aquifer interpretation lies in the evidence, described above, that there is vertical hydraulic continuity between producing zones.

I think it may be possible to devise a multiple-aquifer flow system with barrier boundaries, leakage, and partial penetration that will give a response in the pumped well and observation well that matches the observed data. One could accomplish such a feat by trial and error using a digital simulation

model. However, such a model would contradict the geologic and hydrologic information discussed above. Also, such a model would have little usefulness as it would have no transfer value to other field sites.

The fact, pointed out by Williams, that I treated all the producing zones in the pumped well as one aquifer is an important point. The proposed double-porosity model is probably valid only at the scale of this particular test. By packing off and testing individual production zones, as Williams suggests, the scale of the problem will be changed and effects of aquifer heterogeneity will be magnified. Analysis of pump test data from individual packed-off zones may require that different models be used and may therefore yield different aquifer parameters.

At the present time, based upon the above considerations, I remain convinced that the dual-porosity model for a fissured groundwater reservoir with fracture skin provides the preferred explanation for the well-test data from wells UE-25b #1 and UE-25a #1. The model has the advantage over alternative interpretations of being internally consistent. Two type curves each with identical aquifer parameters (γ , σ , S_p , and W_D) match the well-test data. That is, the observation-well response and the pumped-well response are those predicted by theory. This simple model explains satisfactorily not only the drawdown data but also is in agreement with all the available geological, geophysical, and hydrological data.

Most hydrogeologists will agree that further research on fractured rock is needed. This is especially true as it applies to heterogeneous systems. The work of Williams and his coworkers on the hydrology of hard rock mines is particularly appropriate to this endeavor. The view from within the aquifer may provide a source of much improved understanding of the hydrological behavior of fractured-rock systems.

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Double-Porosity Models for a Fissured Groundwater Reservoir With Fracture Skin

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Theories of flow to a well in a double-porosity groundwater reservoir are modified to incorporate effects of a thin layer of low-permeability material or fracture skin that may be present at fracture-block interfaces as a result of mineral deposition or alteration. The commonly used theory for flow in double-porosity formations that is based upon the assumption of pseudo-steady state block-to-fissure flow is shown to be a special case of the theory presented in this paper. The latter is based on the assumption of transient block-to-fissure flow with fracture skin. Under conditions where fracture skin has a hydraulic conductivity that is less than that of the matrix rock, it may be assumed to impede the interchange of fluid between the fissures and blocks. Resistance to flow at fracture-block interfaces tends to reduce spatial variation of hydraulic head gradients within the blocks. This provides theoretical justification for neglecting the divergence of flow in the blocks as required by the pseudo-steady state flow model. Coupled boundary value problems for flow to a well discharging at a constant rate were solved in the Laplace domain. Both slab-shaped and sphere-shaped blocks were considered, as were effects of well bore storage and well bore skin. Results obtained by numerical inversion were used to construct dimensionless-type curves that were applied to well test data, for a pumped well and for an observation well, from the fractured volcanic rock terrane of the Nevada Test Site.

INTRODUCTION

Fissured or fractured rock formations have been the subject of intensive investigation in recent years. Many productive freshwater-bearing reservoirs, as well as geothermal and petroleum reservoirs, are known to be composed of fractured rock. Also, the search for safe repositories for hazardous wastes has led to studies of low-permeability rock formations, where the major concern is the eventual egress of contaminants to the ecosphere through interconnected conductive fissures.

In order to quantify fluid flow behavior in fractured rock reservoirs, theoretical, laboratory, and field studies have been undertaken. Fractured or fissured reservoirs are complex, heterogeneous, and anisotropic systems. If they are to be treated mathematically, certain idealizations are imperative. Under circumstances where the rock matrix has very low permeability, one idealization has been to assume that flow can occur only in the fractures and not in the matrix. This assumption has been used in studies of discrete fractures and has led to the important finding that discharge through a given fracture is proportional to the cube of the fracture aperture. This "cubic law" appears to hold true for a variety of geometries and for a wide range of stress conditions [Witherspoon *et al.*, 1980; Neuzil and Tracy, 1981; Tsang and Witherspoon, 1981]. The assumption of no flow in the matrix is also used in studies of fractured rock masses where intersecting permeable fractures are treated as a "single-porosity" continuum. Because the fractures often have preferred orientations, apertures, and spacings, quantitative studies may require that hydraulic conductivity be treated as a second-rank tensor rather than a scalar quantity [Snow, 1969]. A general method for field determination of the hydraulic conductivity tensor of a fractured rock has been developed by Hsieh [1983] and is called the "cross-hole" test method.

Studies of fluid flow in a fractured rock mass where fissure

flow is augmented by contributions from the blocks have generally adopted the "double-porosity" concept proposed by Barenblatt *et al.* [1960]. This concept has been used extensively in the petroleum literature. Two approaches to problems of well test analysis have been taken that differ in the manner by which flow from a block to fissure is described. One approach assumes that flow occurs under pseudo-steady state conditions [Warren and Root, 1963] and the other approach assumes that flow occurs under fully transient conditions [for example, Kazemi, 1969]. The assumption of pseudo-steady state block-to-fissure flow is an approximation that has the advantage over the transient flow assumption by providing greater mathematical simplicity; however, it has the disadvantage of ignoring some of the physics of the problem. The assumption of transient block-to-fissure flow is clearly superior from a theoretical standpoint. Interestingly, well test data exist that support both approaches.

It is the purpose of this paper to provide a resolution to this apparent conflict. This is accomplished by using the concept of fracture skin, a thin skin of low-permeability material, deposited on the surfaces of the blocks, that serves to impede the free exchange of fluid between the blocks and fissures. The concept of fracture skin is similar to the idea of fracture "damage" presented by Cinco L. and Samaniego V. [1977] for wells intersecting vertical fractures. The effect of fracture skin in double-porosity systems is to delay flow contributions from the block to fissure and give rise to pressure responses that are similar to those predicted under the assumption of pseudo-steady state flow. By reducing gradients of hydraulic head in the compressible blocks, fracture skin provides theoretical justification for the pseudo-steady state flow approximation used in the Warren and Root [1963] model. To illustrate application of the model an analysis is made of well test data from a pumped well and from an observation well in fractured volcanic rock at the Nevada Test Site.

REVIEW OF DOUBLE-POROSITY MODELS

The concept of double porosity was originally proposed by Barenblatt *et al.* [1969] in order to help quantify flow in fractured rocks. According to this concept, a fractured rock mass

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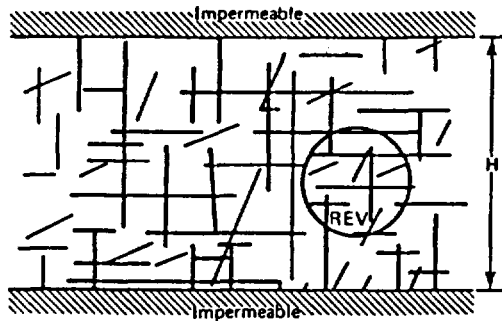


Fig. 1. Schematic diagram of a double-porosity reservoir of thickness H showing a typical representative elementary volume (REV) for the fissure system.

is assumed to consist of two interacting, overlapping continua: a continuum of low-permeability, primary porosity blocks and a continuum of high-permeability, secondary porosity fissures. Such a reservoir is depicted schematically in Figure 1. The primary porosity blocks, where the bulk of the fluid is stored, have hydraulic properties that are generally controlled by depositional and lithification processes. The hydraulic properties of the fissure system are generally the result of thermal stresses and tectonic processes. Blocks and fissures may both have been influenced by chemical precipitation and solution processes or by hydrothermal alteration.

Because of the dual nature of double-porosity reservoirs and because two controlling partial differential equations may be involved, it is helpful to consider two representative elementary volumes (REV's) in describing the system, one for the fissure system and one for the block system. The fissure system REV is assumed to contain a large number of fissures and blocks (see Figure 1) so that adding or subtracting a few blocks from the REV will not substantially alter its hydraulic properties. The fissure system REV is dependent upon the hydraulic properties and geometry of the fissure system and upon the scale of the problem under consideration. The block system REV will not necessarily be the same as the fissure system REV and will depend upon characteristics of the blocks. Under the assumption of pseudo-steady state block-to-fissure flow the two REV's can be taken to be equal to one another. Under the assumption of transient block-to-fissure flow the block-system REV will necessarily be significantly smaller in order to account for the distribution of hydraulic head in a representative block. In this paper the block and fissure continua are assumed to be homogeneous and isotropic with regard to their hydraulic properties. Consequently, the REV's will not vary spatially within the reservoir.

As described by Gringarten [1982], it is possible to define the fissure system hydraulic conductivity as

$$K = K_f V_f \quad (1)$$

and the block system hydraulic conductivity as

$$K' = K_m V_m \quad (2)$$

where K_f and K_m are the hydraulic conductivities of representative fissures and matrix rock, respectively, V_f is the ratio of the total volume of the fissures to the bulk volume of the rock mass (the sum of the volume of the fissures and the volume of the matrix), and V_m is the ratio of the total volume of the matrix rocks to the bulk volume. V_f and V_m sum to unity. Normally, V_m is very close to unity so that $K' \approx K_m$.

Designation of fissure system hydraulic conductivity as shown in (1) has a distinct advantage, as it becomes unnecessary to specify the individual fracture hydraulic conductivity or aperture. In like manner, specific storage of the fissure system can be defined as

$$S_s = S_{sf} V_f \quad (3)$$

and the specific storage of the blocks can be defined as

$$S_s' = S_{sm} V_m \quad (4)$$

where S_{sf} and S_{sm} are the specific storages of representative fissures and matrix rocks, respectively.

The controlling differential equation for flow in the fissure network is assumed to be described by the familiar groundwater diffusion equation with a source term to account for contributions from the matrix rock:

$$K \nabla^2 h = S_s \frac{\partial h}{\partial t} + q_s \quad (5)$$

Similarly, for the matrix rock,

$$K' \nabla^2 h' = S_s' \frac{\partial h'}{\partial t} - q_s \quad (6)$$

The primed quantities that appear in this paper refer to the matrix rock and the unprimed quantities refer to the fissure system. Symbols are defined in the notation section.

Derivations of the groundwater diffusion equation are given in standard texts such as Freeze and Cherry [1979]. The assumptions under which (5) and (6) are derived include the validity of Darcy's law for flow in the fissures and blocks, slightly compressible fluid and rock, constant hydraulic properties, and negligible effects of fluid density gradients.

In their mathematical development for flow to a well in a double-porosity system, Warren and Root [1963] assumed that the left-hand side of (6) was zero. This means, in effect, that spatial variation of hydraulic head gradients or the divergence of flow in the block is ignored. They also assumed that the flux of fluid from blocks to fissures in an REV occurred in response to the difference in the average hydraulic head in the fissures and the average hydraulic head in the blocks. This is the assumption of pseudo-steady state flow and is described mathematically as

$$q_s = -\alpha K'(h' - h) \quad (7)$$

where h' in this case represents a spatial average of hydraulic head within the block and α relates to the geometry of the fissured rock and has the dimensions of inverse area. Barenblatt et al. [1960] made an additional assumption that fluid storage in the fracture network can be ignored, so that the first term on the right-hand side of (5) is negligible compared with the other terms.

Because of the low hydraulic conductivity of the matrix rock, the problem of flow to a production well is usually solved by assuming that fluid enters the well bore only through fissures and not through the matrix rock. By assuming radial flow to a fully penetrating well discharging at a constant rate from an infinitely extensive double-porosity reservoir confined above and below by impermeable formations, the Laplace transform, line source solution for dimensionless drawdown in the fissures can be written as

$$\bar{h}_D = \frac{2}{p} K_\alpha [r_D(p + \bar{q}_D)^{1/2}] \quad (8)$$

where

$$\bar{q}_D = \frac{p}{1/\sigma + p/\lambda} \quad (9)$$

$$\sigma = S_f'/S_s \quad (10)$$

$$\lambda = \alpha(K'/K)r_w^2 \quad (11)$$

\bar{q}_D is the dimensionless flow from block to fissure in Laplace space, p is the Laplace transform variable, and r_w is the radius of the production well. The well radius is introduced in the mathematical development of (8) for convenience; it is not meant to imply here that the well bore has a finite radius. Provided that block geometry is known, the above double-porosity system is completely specified by only two dimensionless parameters, σ and λ .

Dimensionless drawdown is defined as

$$h_D = \frac{4\pi KH}{Q_T} (h_i - h) \quad (12)$$

dimensionless time (inversely related to the Laplace transform variable) is defined as

$$t_D = \frac{Kt}{S_s r_w^2} \quad (13)$$

and dimensionless distance is defined as

$$r_D = r/r_w \quad (14)$$

where H is the reservoir thickness, Q_T is the total well discharge, and h_i is the initial hydraulic head in the reservoir. It should be noted that in the line source solution given by (8), t_D and r_D are not independent of one another and the solution is a function of t_D/r_D^2 .

Although expressed in different notation, (8)–(9) is the same solution as given by Kazemi *et al.* [1969], which is an extension for interference tests of the Warren and Root model. If \bar{q}_D is zero, (8) reduces to the familiar Theis [1935] solution.

Basic to the derivation of (8)–(9) is the pseudo-steady state flow assumption. Models that are based on this approximation show that, in the absence of well bore storage effects, well discharge consists initially of fluid derived primarily from storage in the fissures followed, at late time, by fluid derived primarily from storage in the blocks. At early and late times, drawdown should therefore follow the familiar Theis type curve. During a sufficiently long transition from early to late time, however, drawdown will approach a plateau (see Figure 3, curve C). Well test data exist that support this assumption [for example, Streltsova, 1976, Figure 7; Bourdet and Gringarten, 1980]. Because of such data and because of the relative simplicity of the approach, double-porosity models that

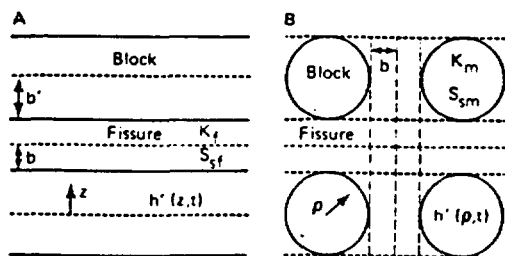


Fig. 2. Geometrical configuration for (a) slab-shaped blocks and (b) sphere-shaped blocks.

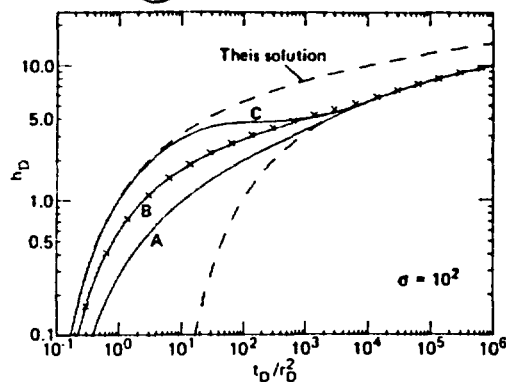


Fig. 3. Line source type curve comparisons for transient flow from sphere-shaped blocks (curve A, $r_D^2/\gamma^2 = 0.01$) and slab-shaped blocks (curve B, $r_D^2/\gamma^2 = 0.01$) and pseudo-steady state from slab-shaped blocks (curve C, $r_D^2/\gamma^2 = 0.03$). Also shown (crosses) are values for sphere-shaped blocks, assuming that $(r_D^2/\gamma^2) = 0.01/9$.

invoke the assumption of pseudo-steady state flow have received much attention in the literature [see Gringarten, 1982]. The assumption of pseudo-steady state flow, however, does not have a firm theoretical justification.

In order to account for transient flow from blocks to fissures, it is necessary to specify block geometry. Kazemi [1969] assumed transient block-to-fissure flow and used a finite difference model to simulate flow to a well. He assumed the fractured rock mass could be idealized as alternating layers (slabs) of blocks and fissures where the thickness of the blocks and the aperture of the fissures represent average fracture spacings and apertures. By simulating flow in the blocks as well as in the fissures, he was able to account for the variation that must occur in the hydraulic head in the blocks. This problem was later solved analytically by Boulton and Streltsova [1977] and Najurieta [1980] by assuming strictly one-dimensional flow in the block perpendicular to the block-fissure interface.

In the notation of this paper, and referring to the schematic diagram in Figure 2a, the Laplace transform line source solution for dimensionless drawdown in the fissures using slab-shaped blocks is (8), where

$$\bar{q}_D = \gamma^2 m \tanh(m) \quad (15)$$

$$m = (\sigma p)^{1/2} / \gamma \quad (16)$$

$$\gamma = \frac{r_w}{b'} \left(\frac{K'}{K} \right)^{1/2} \quad (17)$$

and b' is the average half thickness of the slab.

Type curves for this model differ from those obtained under the pseudo-steady state flow approximation during the period of transition from early to late time. Under the assumption of transient flow there is a gradual transition resulting in early departure from the initial Theis type curve (see Figure 3, curve B). Comparisons of the type curves obtained for each of these two assumptions are given by Streltsova [1983], Deruyck *et al.* [1982], and Gringarten [1982]. During a transition period of sufficiently long duration (large σ) a semilogarithmic plot of drawdown versus time may show a straight line with a slope of one half the slope that occurs during the late time period (see Figure 6c for $S_f = 0$). Cinco L. and Samaniego V. [1982] pointed out that many data sets published in the literature support the assumption of transient flow from blocks to fissures [for example, Warren and Root, 1963; Najurieta, 1980].

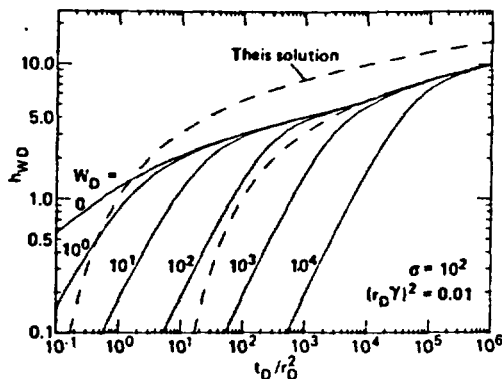


Fig. 4. Type curves showing effects of well bore storage in production well, assuming transient flow from slab-shaped blocks.

Analytical studies have not been restricted to flow within and adjacent to horizontal slabs. Other geometries, such as cylinders or spheres, have been used to idealize columnar joints, cubes, or parallelepipeds [de Swaan O., 1976; Kućuk and Sawyer, 1980; see also Deruyck et al., 1982]. In the notation of this paper, and referring to the schematic diagram in Figure 2b, the Laplace transform line source solution for dimensionless drawdown in the fissures using sphere-shaped blocks is (8), where

$$\bar{q}_D = 3\gamma^2[m \coth(m) - 1] \quad (18)$$

and b' now represents the average radius of the sphere-shaped blocks.

Comparisons of the type curves resulting from the different models (slabs, cylinders, and spheres) for transient flow from blocks to fissures show that they are all very nearly the same. Evaluation of the parameters will give different results, however, as can be seen by comparing (15) and (18). For large m (early time), $\tanh(m)$ in (15) approaches unity and $\bar{q}_D \approx \gamma(\sigma\rho)^{1/2}$. From (18) it can be seen that $\bar{q}_D \approx 3\gamma(\sigma\rho)^{1/2}$. It is therefore not surprising that computations show that if the hydraulic conductivity of the slab-shaped block is taken to be 9 times that of the sphere-shaped blocks, the resulting type curves will be indistinguishable at early time.

Figure 3 shows a comparison of line-source type curves for sphere-shaped blocks (curve A) and slab-shaped blocks (curve B) using the indicated values of the parameters σ and γ . Figure 3 (curve C) shows the corresponding type curve in the case of pseudo-steady state flow from fissure to block where, for slab-shaped blocks $\alpha = 3/(b')^2$ [see Warren and Root, 1963, Appendix A]. The effect of reducing hydraulic conductivity in sphere-shaped blocks by a factor of 9 is also illustrated in Figure 3. Although it is not apparent in Figure 3, there is a slight difference in the shape of the type curves during the time of transition to the late time Theis curve for sphere-shaped blocks compared with slab-shaped blocks. This difference becomes more apparent as $(r_D\gamma)^2$ is increased. The curves are bounded on the left by the Theis type curve and on the right by the Theis type curve displaced to the right by a factor of $1 + \sigma$. As $(r_D\gamma)^2$ or $r_D^2\lambda$ is reduced, the type curves A, B, and C will move upward in Figure 3. Comparison of (9) and (15) or (18) shows that the equations become identical at late time.

The type curves in Figure 3 were obtained through numerical inversion of (8), using (9), (15), and (18). This was accomplished by use of the Stehfest [1970] algorithm, which is easy to use and has proven to be exceptionally accurate for a large

class of groundwater and petroleum engineering problems. Moench and Ogata [1984] provided a discussion of the algorithm and applied it to other problems of groundwater hydrology.

In the analysis of production well data and sometimes for the analysis of nearby observation well data it may be necessary to consider effects of well bore storage and well bore skin [Ramey and Agarwal, 1972; Sandel et al., 1978; Chu et al., 1980]. The Laplace transform solution for a production well in a fractured reservoir assuming pseudo-steady state flow from blocks to fissures was given by Mavor and Cinco L. [1979]. In the notation of this paper the solution is written as

$$\bar{h}_{wD} = \frac{2[K_0(x) + xS_wK_1(x)]}{p\{pW_D[K_0(x) + xS_wK_1(x)] + xK_1(x)\}} \quad (19)$$

where

$$x = (p + \bar{q}_D)^{1/2}$$

For an observation well the solution is

$$\bar{h}_D = \frac{2K_0(r_Dx)}{p\{pW_D[K_0(x) + xS_wK_1(x)] + xK_1(x)\}} \quad (20)$$

S_w is the dimensionless well bore skin commonly used in petroleum engineering. The skin is assumed to be infinitesimally thin, incompressible, and invariant for the duration of the well test. W_D is the dimensionless well bore storage, defined in this paper as

$$W_D = \frac{C}{2\pi r_w^2 S_w H} \quad (21)$$

where C is the well bore storage and is assumed to be constant for the duration of the test. Ramey and Agarwal [1972] pointed out that the effects of well bore storage can be by virtue of changing liquid level in the well casing or by virtue of compressibility. Effects of well bore storage are greatest when due to changing liquid level. In this case, $C = \pi r_c^2$, where r_c is the radius of the well casing in the region of the changing liquid level. If the effects are due to compressibility of the liquid and of the well itself in a pressurized well [see Neuzil, 1982; Bredehoeft and Papadopoulos, 1980], $C = V_w \rho_w g C_{obs}$, where V_w is the volume of liquid in the pressurized section, ρ_w is the liquid density, g is the acceleration of gravity, and C_{obs} is the observed compressibility of the combined fluid-well system.

The solutions given by (19) and (20) apply equally well when \bar{q}_D is defined as in (9), (15), or (18). As with the line source solution (8) the above equations can be inverted numerically by using the Stehfest algorithm. If S_w and \bar{q}_D are zero, (19) and (20) reduce to the large-diameter well equation of Papadopoulos [1967] and Papadopoulos and Cooper [1967]. If S_w and W_D are zero, (19) reduces to the solution presented by Warren and Root [1963] for a production well of finite diameter without well bore storage or skin.

Figure 4 illustrates typical type curves for a production well, showing the effect of well bore storage by using (19) together with (15) for slab-shaped blocks. Note that with sufficiently

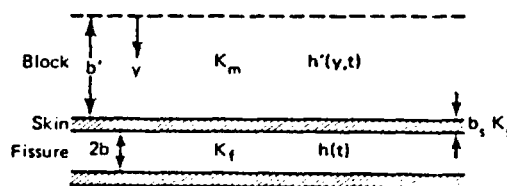


Fig. 5. Schematic diagram of a block and fissure with fracture skin.

large dimensionless well bore storage the transition from early time to late time is obliterated.

FISSURED RESERVOIRS WITH FRACTURE SKIN

It is difficult to justify, on theoretical grounds alone, use of the pseudo-steady state block-to-fissure flow model. In response to changes in hydraulic head in a fissure there will be induced in the blocks, near the block-fissure interface, a curvature in the hydraulic head distribution that will increase with decreasing block hydraulic conductivity. This means that during the period of transition from early to late time, when flow from the block has a profound influence on the draw-down in the fissure system, neglect of the left-hand side of (6) is not justifiable. For this reason the transient block-to-fissure flow model which does not neglect the divergence of flow in the blocks has a distinct theoretical advantage. On the other hand, it has been pointed out that well test data exist that support the pseudo-steady state flow model.

In this section the transient flow model is modified by introducing the concept of fracture skin. This not only provides theoretical justification for the use of the pseudo-steady state flow model but also allows for evaluation of the fracture skin parameter and, if the data permit, block hydraulic conductivity. It is shown that if the fracture skin parameter is sufficiently large (low fracture skin hydraulic conductivity), most of the change in hydraulic head that occurs in a block occurs across the fracture skin. Consequently, changes in the gradients of hydraulic head in the block become small, thus justifying the assumption in the pseudo-steady state flow model that the divergence of flow in the block is negligible. By suitable modification of the parameter α in (11), so that it accounts for the hydraulic conductivity of the fracture skin rather than the block hydraulic conductivity, it is shown that the transient flow model reduces to the pseudo-steady state flow model if the fracture skin parameter is sufficiently large.

Figure 5 shows a schematic diagram of a thin but finite thickness skin on the surface of a hypothetical block in a double-porosity reservoir. It is assumed that the skin possesses negligible storage capacity. Flux of fluid from the block to the fissure is assumed to be perpendicular to the interface and to obey Darcy's law. Hence, by continuity, at the fissure-block interface,

$$K_s \frac{[(h')_{y=b} - h]}{b_s} = -K_m \left(\frac{\partial h'}{\partial y} \right)_{y=b} \quad (22)$$

where $(h')_{y=b}$ is the hydraulic head in the block at the skin

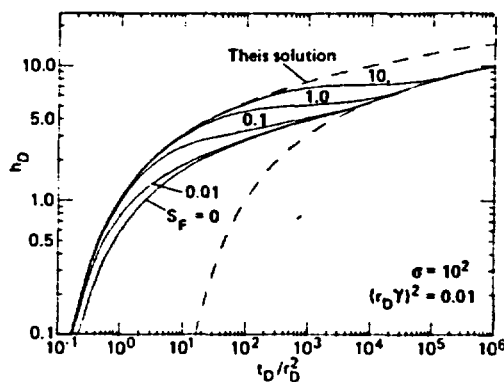


Fig. 6a. Line source, type curves for transient flow from slab-shaped blocks with fracture skin.

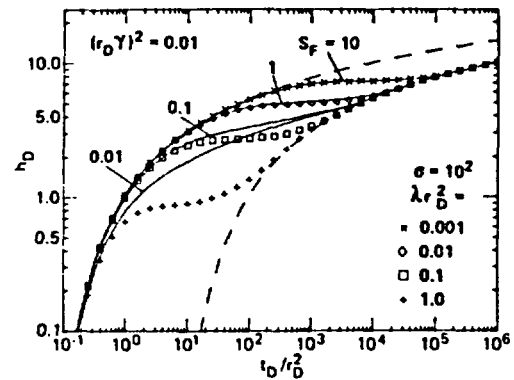


Fig. 6b. Comparison of Figure 6a with corresponding type curves for the pseudo-steady state flow model.

surface, K_s is the hydraulic conductivity of the skin, and b_s is the thickness of the skin. In what follows it is assumed that $b_s \ll b'$. It is not necessary that $K_s < K_m$. Equation (22) derives from heat flow theory [see Carslaw and Jaeger, 1959, p. 20].

Fracture skin may occur in naturally fractured reservoirs as a result of mineral alteration or deposition. Such alteration products and minerals are commonly seen lining or filling fractures in surface outcrops or well core samples. Their presence is probably due to the fact that interconnected fissures serve as primary conduits for mineral-charged water circulating naturally in groundwater and geothermal reservoirs. Such fluids may not be in chemical or thermodynamic equilibrium with the wall rock and hence may alter or deposit minerals there. The particular products of hydrothermal alteration or deposition depend upon physical and chemical parameters that vary from place to place. Specific types of such products that occur in geothermal systems were described by Ellis and Mahon [1977].

In a study of the Salton Sea geothermal field, Tewley [1977, p. 32] observed open fractures that "... manifest themselves as planar surfaces coated with a thin layer of fracture-filling mineral." Tewley cited evidence that filled or sealed fractures can be reopened by tectonic stresses and natural hydraulic fracturing. Many fractures occurring in the volcanic and carbonate rocks of the Nevada Test Site, Nevada are reported to be filled or partially filled with various mineral assemblages [Young, 1972; Blankennagel and Weir, 1973; Winograd and Thordarson, 1975; Spengler et al., 1979]. Excellent photographs of cores with partially filled fractures are presented by

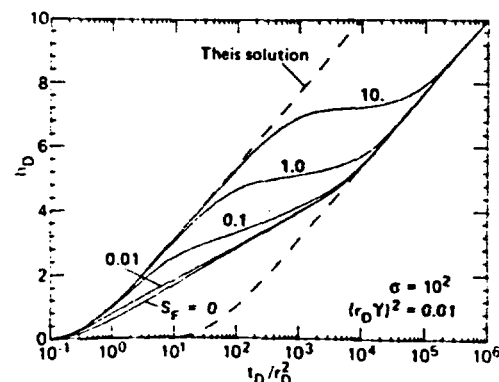


Fig. 6c. Semilogarithmic plot of Figure 6a.

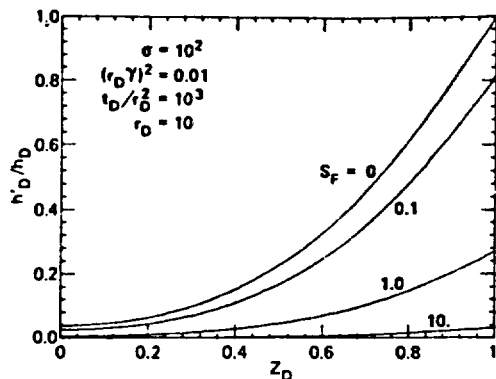


Fig. 7a. Distribution of hydraulic head for various values of S_F in slab-shaped blocks at the indicated values of dimensionless time and distance.

Davidson and Snowdon [1978] for a carbonate rock gas reservoir in northwestern Canada. It is reasonable to conjecture that such mineral deposits are common in nature and that many serve to plug pores of primary porosity blocks at fissure block interfaces.

Transient Block-to-Fissure Flow

In this section the boundary value problems and Laplace transform solutions are given for transient block-to-fissure flow with fracture skin. Derivations for slab-shaped blocks and sphere-shaped blocks are given in Appendices A and B; see Figures 2 and 5 and the notation section for definition of symbols. The generalized coordinate y in Figure 5 becomes z for slab-shaped blocks and ρ for sphere-shaped blocks in what follows.

The controlling differential equation for flow in a fissure is assumed to be the radial diffusion equation with a source term,

$$K_f \frac{\partial^2 h}{\partial r^2} + K_f \frac{1}{r} \frac{\partial h}{\partial r} = S_{if} \frac{\partial h}{\partial t} + q \quad r_w \leq r \quad (23a)$$

where

$$q = \frac{K_m}{b} \left(\frac{\partial h'}{\partial y} \right)_{y=b'} \quad (23b)$$

By substituting (1) and (2) into these expressions, the controlling equation for the fissure network becomes

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S_f}{K} \frac{\partial h}{\partial t} + \frac{qV_f}{K} \quad r_w \leq r \quad (24)$$

where

$$q = \frac{K'}{V_m b} \left(\frac{\partial h'}{\partial y} \right)_{y=b'} \quad (25)$$

For slab-shaped blocks,

$$V_f = \frac{b}{b' + b} \approx \frac{b}{b'} \quad (26)$$

$$V_m = \frac{b'}{b' + b} \approx 1 \quad (27)$$

hence

$$\frac{qV_f}{K} = \frac{K'}{Kb'} \left(\frac{\partial h'}{\partial z} \right)_{z=b'} \quad (28)$$

For sphere-shaped blocks,

$$V_f \approx \frac{4\pi(b')^2 b}{4/3\pi(b' + b)^3} \approx \frac{3b}{b'} \quad (29)$$

$$V_m \approx \frac{4/3\pi(b')^3}{4/3\pi(b' + b)^3} \approx 1 \quad (30)$$

hence

$$\frac{qV_f}{K} = \frac{3K'}{Kb'} \left(\frac{\partial h'}{\partial \rho} \right)_{\rho=b'} \quad (31)$$

The initial condition is

$$h = h_i \quad r_w \leq r \quad (32)$$

The boundary conditions are

$$h = h_i \quad r = \infty \quad (33)$$

$$4\pi r_w K_f b n \left(\frac{\partial h}{\partial r} \right)_{r=r_w} = nQ_f + C \frac{\partial h_w}{\partial t} \quad r = r_w \quad (34)$$

where

$$h_w = h - r_w S_w \frac{\partial h}{\partial r} \quad (35)$$

Q_f is the constant discharge from a well intersecting a single fissure of aperture $2b$, and n is the number of producing fissures. Replacing n by $H/2b'$ in (34) and assuming that $b' + b \approx b'$, the well bore boundary condition becomes

$$2\pi r_w K H \left(\frac{\partial h}{\partial r} \right)_{r=r_w} = Q_f + C \frac{\partial h_w}{\partial t} \quad (36)$$

where

$$Q_f = \frac{H Q_f}{2b'} \quad (37)$$

Slab-shaped blocks. Referring to Figure 2a, the governing equation for slab-shaped blocks is the planar diffusion equation

$$\frac{\partial^2 h'}{\partial z^2} = \frac{S_f}{K'} \frac{\partial h'}{\partial t} \quad 0 < z \leq b' \quad (38)$$

The initial condition is

$$h' = h_i \quad 0 < z \leq b' \quad (39)$$

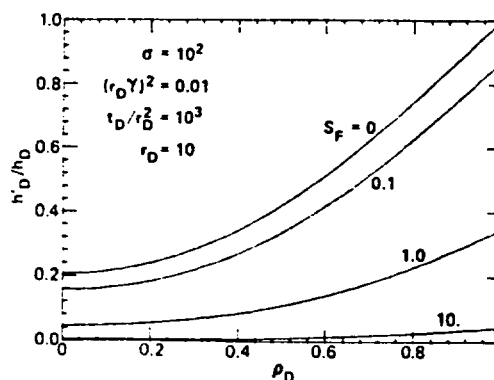


Fig. 7b. Distribution of hydraulic head for various values of S_F in sphere-shaped blocks at the indicated values of dimensionless time and distance.

The boundary conditions are

$$\partial h' / \partial z = 0 \quad z = 0 \quad (40)$$

$$h' = h - b'S_F(\partial h' / \partial z) \quad z = b' \quad (41)$$

where

$$S_F = K'b_s / K_p b' \quad (42)$$

Equation (41) is a rearrangement of (22) with $K' \approx K_m$. S_F is termed the dimensionless fracture skin in this paper.

In addition to the dimensionless parameters in (12)–(14) the following are defined:

$$h_{wD} = \frac{4\pi KH}{Q_T} (h_i - h_w) \quad (43)$$

$$h_{D'} = \frac{4\pi KH}{Q_T} (h_i - h') \quad (44)$$

$$z_D = z/b' \quad (45)$$

The above coupled boundary value problem for slab-shaped blocks is put in dimensionless form by incorporating the dimensionless parameters and is solved in Laplace space (see Appendix A). The Laplace transform solutions for dimensionless hydraulic head in the production well and in the fissure network is given by (19) and (20) with

$$\bar{q}_D = \frac{\gamma^2 m \tanh(m)}{1 + S_F m \tanh(m)} \quad (46)$$

The Laplace transform solution for dimensionless drawdown within a representative slab-shaped block is

$$\bar{h}_{D'} = \frac{\bar{h}_D}{[1 + S_F m \tanh(m)]} \frac{\cosh(z_D m)}{\cosh(m)} \quad (47)$$

For a line source the Laplace transform solution is given by (8) with \bar{q}_D as defined in (46). The line source solution is not derived in this paper but can be obtained in a manner similar to that outlined in Appendix A by having $r \rightarrow 0$ and by making appropriate changes in the inner boundary condition (36).

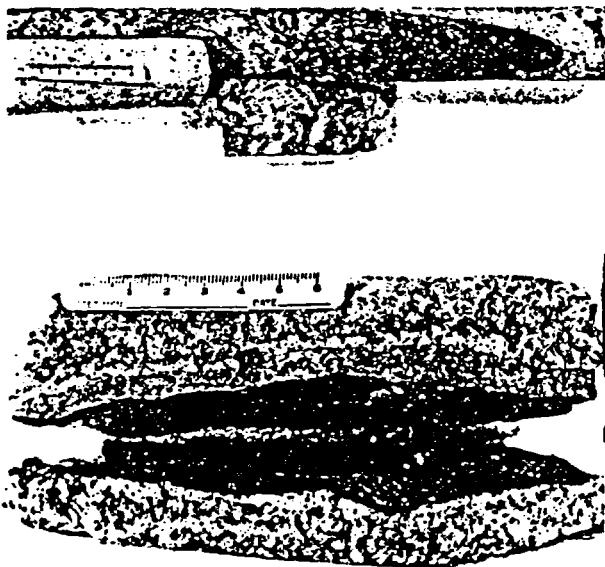


Fig. 8. Typical mineral-filled fractures. (Top) Core from well UE-25a #1 at a depth of 676 m. (Bottom) Core from well UE-25b #1 at a depth of 612 m.

TABLE 1. Well Characteristics

	Pumped Well	Observation Well
Well Number	UE-25b #1	UE-25a #1
Ground elevation, m	1198.7	1198.7
Total depth, m	1219.2	762.2
Approximate depth to static water level, m	470	470
Well diameter below static water level, m	0.22	0.075
Drilling medium	air, detergent, and water	bentonite mud
Bottom of casing, m	518	none
Perforations, m	477–501	none

Sphere-shaped blocks. Referring to Figure 2b, the governing equation for sphere-shaped blocks is [see Carslaw and Jaeger, 1959, p. 230]

$$\frac{\partial^2(\rho h')}{\partial \rho^2} = \frac{S_F}{K'} \frac{\partial(\rho h')}{\partial t} \quad 0 < \rho \leq b' \quad (48)$$

The initial condition is

$$h' = h_i \quad 0 < \rho \leq b' \quad (49)$$

The boundary conditions are

$$h' \text{ finite} \quad \rho = 0 \quad (50)$$

$$h' = h - b'S_F(\partial h' / \partial \rho) \quad \rho = b' \quad (51)$$

With one additional dimensionless parameter defined as

$$\rho_D = \rho/b' \quad (52)$$

the above coupled boundary-value problem is put in dimensionless form and solved in Laplace space (see Appendix B). The Laplace transform solutions (19) and (20) also apply for

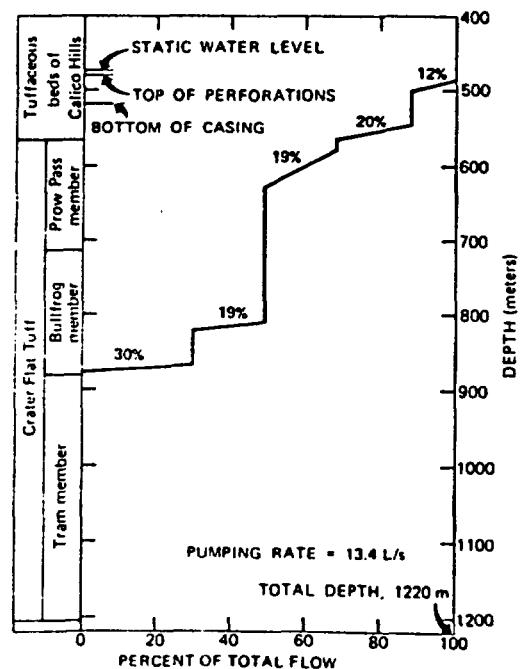


Fig. 9. Bore hole survey showing percent of total flow versus depth and stratigraphic units of Miocene age in pumped well UE-25b #1 (adapted from Lahoud et al.).

TABLE 2. Drawdown Data for Test 3

t , min	$h_i - h_e$, m
<i>Pumped Well</i>	
0.05	2.513
0.1	3.769
0.15	4.583
0.2	4.858
0.25	5.003
0.3	5.119
0.35	5.230
0.4	5.390
0.45	5.542
0.5	5.690
0.6	5.960
0.7	6.19
0.8	6.42
0.9	6.59
1.0	6.74
1.2	6.96
1.4	7.17
1.6	7.33
1.8	7.45
2.0	7.56
2.5	7.76
3.0	7.93
3.5	8.03
4.0	8.12
5.0	8.24
6.0	8.32
7.0	8.41
8.0	8.46
9.0	8.54
10.0	8.62
12.0	8.67
14.0	8.70
16.0	8.74
18.0	8.76
20.0	8.77
25.0	8.81
30.0	8.84
35.0	8.84
40.0	8.86
50.0	8.86
60.0	8.90
70.0	8.91
80.0	8.92
90.0	8.93
100.0	8.95
120.0	8.97
140.0	8.98
160.0	8.99
180.0	9.00
200.0	9.02
240.0	9.04
300.0	9.07
400.0	9.11
500.0	9.14
600.0	9.17
700.0	9.18
800.0	9.21
900.0	9.25
1000.0	9.30
1200.0	9.44
1400.0	9.55
1600.0	9.64
1800.0	9.74
2000.0	9.78
2200.0	9.80
2400.0	9.84
2600.0	9.93
2800.0	10.03
3000.0	10.08
3500.0	10.26

TABLE 2. (Continued)

t , min	$h_e - h_e$, m
<i>Pumped Well (Continued)</i>	
4000.0	10.30
4200.0	10.41
<i>Observation Well</i>	
0.5	0.002
0.6	0.002
0.7	0.002
0.8	0.002
0.9	0.002
1.0	0.005
1.2	0.005
1.4	0.007
1.6	0.007
1.8	0.012
2.0	0.015
2.2	0.015
2.4	0.020
2.6	0.022
2.8	0.025
3.0	0.027
3.5	0.037
4.0	0.045
4.5	0.052
5.0	0.059
5.5	0.069
6.0	0.079
7.0	0.097
8.0	0.116
9.0	0.134
10.0	0.151
12.0	0.186
14.0	0.213
16.0	0.238
18.0	0.260
20.0	0.285
25.0	0.320
30.0	0.342
35.0	0.359
40.0	0.374
45.0	0.384
50.0	0.392
60.0	0.401
70.0	0.411
120.0	0.434
140.0	0.439
160.0	0.444
180.0	0.451
200.0	0.453
240.0	0.461
280.0	0.468
300.0	0.471
340.0	0.478
400.0	0.491
440.0	0.498
500.0	0.506
600.0	0.518
700.0	0.525
800.0	0.528
900.0	0.528
1000.0	0.538
1200.0	0.563
1400.0	0.577
1600.0	0.577
1800.0	0.577
2000.0	0.590
2300.0	0.587
2700.0	0.615
3000.0	0.615
3500.0	0.627
3680.0	0.639

sphere-shaped blocks, with

$$\bar{q}_D = \frac{3\gamma^2[m \coth(m) - 1]}{\{1 + S_F[m \coth(m) - 1]\}} \quad (53)$$

The Laplace transform solution for dimensionless drawdown at any point within a representative sphere-shaped block is

$$\bar{h}_D = \frac{\bar{h}_D}{\rho_D \{1 + S_F[m \coth(m) - 1]\}} \frac{\sinh(\rho_D m)}{\sinh(m)} \quad (54)$$

As for slab-shaped blocks the line source Laplace transform solution is given by (8) with \bar{q}_D now defined by (53).

Pseudo-Steady State Block-to-Fissure Flow

In this section the approach of Warren and Root [1963] and of other investigators that invokes the pseudo-steady state flow assumption is modified to incorporate fracture skin. When this is done the hydraulic conductivity of the block is replaced by the hydraulic conductivity of the fracture skin, resulting in a revised definition of λ . Evaluation of λ from a well test will then yield a value of the ratio K_s/b_s rather than hydraulic conductivity K' . Derivations of the Laplace transform solutions are given in Appendix C.

Substituting (22) and (25) into (24) and comparing the result with (5), it is seen that (7) can be written as

$$q_s = -V_f \frac{K_s (h' - h)}{b_s} \quad (55)$$

where $h' - h$ is the head differential across the skin of thickness b_s and V_m has been assumed equal to unity. In this development it is not necessary to specify the location of h' in (22) as $(h')_{r=b_s}$ because it is assumed that h' is uniform throughout the block at any instant of time. Accordingly, it is now admissible to neglect the divergence of flow in the blocks, and (6) becomes

$$q_s = S_s' \frac{\partial h'}{\partial t} \quad (56)$$

Combining (5), (55), and (56), it is shown in Appendix C that \bar{q}_D is given by (9) with

$$\lambda = \frac{r_w^2 K_s V_f}{K b b_s} = \frac{\gamma^2 V_f b'}{S_F b} \quad (57)$$

Thus, for slab-shaped blocks,

$$\bar{q}_D = \frac{\gamma^2 m^2}{1 + S_F m^2} \quad (58)$$

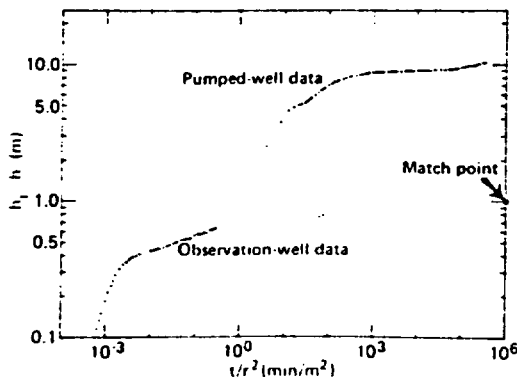


Fig. 10. Drawdown data for test 3 in well UE-25b #1 ($r_w = 0.11$ m) and well UE-25a #1 ($r = 110$ m).

TABLE 3. Match Point and Dimensionless Parameters

Parameter	Value
h_D	1.4
$h_i - h$	1.0 m
W_D	1×10^3
γ^2	1.8×10^{-6}
t_D/r_D^2	4×10^8
t/r^2	10^6 min m^{-2}
σ	2×10^2
S_F	1.0

and for sphere-shaped blocks

$$\bar{q}_D = \frac{3\gamma^2 m^2}{3 + S_F m^2} \quad (59)$$

(58) and (59) can be used in (8), (19), or (20) to obtain the necessary type curves.

Comparison of the Laplace transform expressions (46) and (53) for the transient flow assumption with the expressions (58) and (59) for the pseudo-steady state flow assumption shows that they become equivalent for certain ranges of the indicated parameters. For large S_F and large m (short time), (46) and (58) become

$$\bar{q}_D = \gamma^2 / S_F \quad (60)$$

and (53) and (59) become

$$\bar{q}_D = 3\gamma^2 / S_F \quad (61)$$

For large S_F and small m (long time), (46) and (53) become identical to (58) and (59). This occurs when $m^2 \leq 0.1$ or, in real space, when the following criterion holds:

$$t_D \geq 10 \frac{\sigma}{\gamma^2} \quad (62)$$

Alternatively, the criterion can be written as

$$t \geq 10 \frac{(b')^2}{K'/S_s'} \quad (63)$$

Hence, for the criterion (62) to hold over a broad time range, block diffusivity (K'/S_s') must be large and/or block dimensions must be small. For a given block size, (46) and (53) will reduce to (58) and (59) only if S_F is large and block diffusivity is large or, alternatively, if the time of interest is large.

When S_F is zero, (46) and (53) reduce to (15) and (18). For the pseudo-steady state flow assumption, however, zero S_F implies immediate and complete drainage from the blocks due to any decline in hydraulic head in the fissures. In this instance

$$\bar{q}_D = \gamma^2 m^2 = \sigma p \quad (64)$$

All expressions for \bar{q}_D approach that given in (64) when m is sufficiently small.

Figure 6a shows line source type curves using (8) and (46) for different values of S_F and the indicated values of σ and $(r_D \gamma)^2$. Figure 6b shows the same type curves compared with the corresponding curves using (9) or (58), where λ is obtained by dividing γ^2 by S_F . As S_F increases, it can be seen that the type curves obtained by using (46) approach those using (58). A similar correspondence can be shown using sphere-shaped blocks.

As was pointed out in the last section, where $S_F = 0$, a

TABLE 4. Calculated Hydraulic Properties

Parameter	Value
K	$1 \times 10^{-3} \text{ m s}^{-1}$
S_f	$1.5 \times 10^{-6} \text{ m}^{-1}$
K_f/b_f	$5 \times 10^{-8} \text{ s}^{-1}$
K'	$2 \times 10^{-6} \text{ m s}^{-1}$
S_f'	$3 \times 10^{-4} \text{ m}^{-1}$

semilogarithmic plot of drawdown versus time during the transition from early to late time may show a straight line with a slope equal to one half that of the late time semilogarithmic straight line. This is shown in Figure 6c, which is the same as Figure 6a plotted in semilogarithmic coordinates. In the presence of fracture skin there may appear to be a semilogarithmic straight line, but its slope may be very much less than one half the late time slope and, if not properly interpreted, may give rise to an erroneously large computed value of fissure system hydraulic conductivity.

Figures 7a and 7b show the distribution of hydraulic head, computed using (47) and (54), for different values of S_f in slab-shaped and sphere-shaped blocks at a given time and given radial distance from the production well. It is apparent in both instances that the distribution of hydraulic head in the block becomes uniform when S_f is sufficiently large. This then provides the necessary theoretical justification for neglecting the divergence of flow in (6) and for use of the assumption of pseudo-steady state flow from block to fissure.

CASE STUDY

In order to assist the U.S. Department of Energy in determining the suitability of underground repositories for storage of waste radionuclides, the U.S. Geological Survey has been conducting intensive investigations to characterize the hydraulic properties of the Tertiary volcanic rocks in the vicinity of Yucca Mountain at the Nevada Test Site. Geologic, geophysical, petrologic, and hydrologic studies have been made, and numbers of wells have been drilled, cored, and tested. Scott *et al.* [1983] gave details of the geologic setting, structure, stratigraphy, rock physical properties, and fault and fracture systems in the area of interest known as Drill Hole Wash, located on the east flank of Yucca Mountain. For the present study, well test data were made available (J. Robison, personal communication, 1982) from a pumped well (UE-25b #1) and an observation well (UE-25a #1). These wells were drilled through thick sequences of fractures and faulted nonwelded to densely welded rhyolitic, ash flow, and bedded tuffs to depths below the water table which is located at a depth of 470 m. Circulation losses that occurred during coring of the highly fractured intervals suggest that many fractures are open and interconnected [Spengler *et al.*, 1979]. Core samples reveal that most of the fractures are steeply dipping and coated with deposits of silica, manganese and iron oxides, and calcite. Down-hole televiwer records show that water-producing zones are associated with low-angle fractures (R. G. Lahoud *et al.*, unpublished manuscript, 1984; hereinafter Lahoud *et al.*). Figure 8 shows photographs of typical mineral-filled fractures in cores taken from the observation well and pumped well at depths of 676 m and 612 m, respectively. Rock at these depths was reported to be partially to moderately welded, devitrified ash flow tuff belonging to the Prow Pass Member of Crater Flat Tuff of Miocene age [Spengler *et al.*, 1979]. The minerals deposited in these fractures probably include MnO_2 and silica.

Further details regarding the fractures, rocks, and minerals encountered in these boreholes were provided by Spengler *et al.* [1979] and Caporuscio *et al.* [1982]. Lahoud *et al.* provided specific information about the pumped well including construction, geophysical logs, borehole flow surveys, results of packer and pumping tests, water chemistry, fracture analysis, and results of laboratory tests performed on rock cores. Although both wells deviate from the true vertical, they appear to deviate by about the same magnitude and direction. The contacts between the producing formations occur at about the same depths in the pumped well and in the observation well. For these reasons, no corrections were applied in calculating the distance between zones of entry in the two wells. Pertinent information regarding the wells is given in Table 1. The results of a bore hole flow survey conducted in the pumped well are shown in Figure 9. There appear to be five major zones of entry over a depth interval of about 400 m, which is taken to be the reservoir thickness for the purpose of this analysis. The lack of significant head differences in packed-off intervals under static conditions suggests good hydraulic connection between zones (Lahoud *et al.*).

Table 2 shows the drawdown data for test 3 that are analyzed in this case study. Figure 10 shows a logarithmic plot of h versus t/r^2 where $r_w = 0.11$ m for the pumped well and $r = 110$ m for the observation well. Test 3 was the third test in a series of three that were conducted on well UE-25b #1 in October 1981. Test 1 was run as a single-well test with a small capacity pump. The borehole flow survey was conducted at the end of this test. Test 2 was a long test (9 days), but the flow rate was not constant and the drawdown curves show the effect of continuing well development (D. Lohmeyer, personal communication, 1982). During the final two days of test 2 the well was pumped at the same rate as test 3. It is encouraging to note that recovery data from test 2 are nearly identical to drawdown data for test 3, except for times less than 6 minutes, for both the pumping well and the observation well. Test 3 was pumped at a constant rate of 35.8 L/s for nearly 3 days. At the end of this test the pumped water had a temperature of 35°C at the well head. Plots of drawdown and recovery for these various tests were given by Lahoud *et al.*

It was decided to analyze the drawdown data by assuming slab-shaped blocks. This was due to the scale of the problem and the observation that the distance between the two wells and the average distance between the zones of water entry shown in Figure 9 are of the same magnitude, about 100 m. In

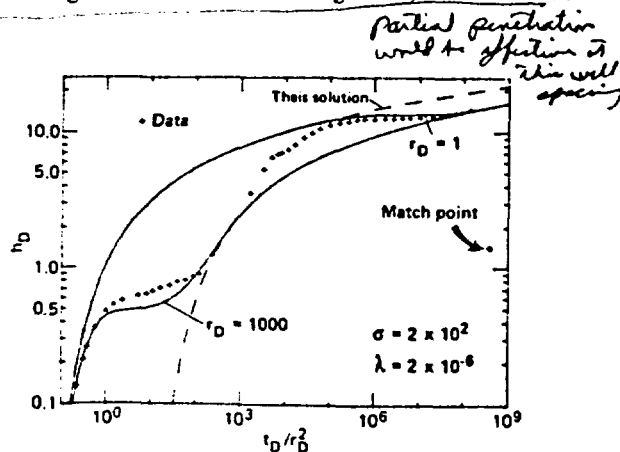


Fig. 11. Comparison of drawdown data to line source type curves using the pseudo-steady state block-to-fissure flow model and the indicated parameters.

the absence of detailed description of the fracture network, the author believes that more closely spaced water entries are needed to justify the use of sphere-shaped blocks.

Six dimensionless parameters (see Table 3) can be obtained by a type curve match of the data in Figure 10. From these, six reservoir parameters can be estimated. Fortunately, these dimensionless parameters tend to influence the character of different parts of the type curves. This makes it possible to obtain a set of parameters that is unique to the problem at hand. Nevertheless, it is helpful to use a systematic approach in attempting to obtain a match. In the analysis of the data in Figure 10 the parameters σ and λ were first estimated by applying the pseudo-steady state block-to-fissure flow model. This involved numerical inversions of the Laplace transform solution for a line source given by (8) and (9) using different values of σ and λ . A rough type curve match was obtained by varying σ to obtain a match in the horizontal direction and varying λ to obtain a match in the vertical direction. As shown in Figure 11, a single value of σ and λ gives a rough match to both the pumped well data and the observation well data. To avoid confusion, the data in Figure 11 (and in Figures 12 and 13) are plotted without showing the ordinate and abscissa. The data are plotted at the same scale shown in Figure 10, with the match point indicated on the figures.

The type curve match shown in Figure 11 under the assumption of pseudo-steady state block-to-fissure flow might be considered satisfactory. Such a match is not possible under the assumption of transient block-to-fissure flow without the influences of fracture skin. Fortunately, because the dimensionless fracture skin factor is not so large as to obliterate the transient character of the flow from the blocks, it is possible to use the theory presented in this paper to obtain separate values of γ^2 and S_F . This will allow for evaluation of K' .

This is done as illustrated in Figure 12 by using (8) and (46) and holding the ratio γ^2/S_F constant (equal to λ) while varying S_F . Of the three type curves for $r_D = 1000$ shown in Figure 12, $S_F = 1$ appears to provide the best agreement with the observation-well data. At the resolution of this plot, pumped well data is not diagnostic of S_F . When an expanded scale is used, the pumped well data can be shown to match type curves for $S_F = 1$ or greater.

The final match between data and type curves is shown in Figure 13. Laplace transform solutions that include effects of well bore storage, given by (19) and (20), were used together with the expression for \bar{q}_D given by (46) for slab-shaped blocks. Well bore skin was assumed to be negligible. This assumption

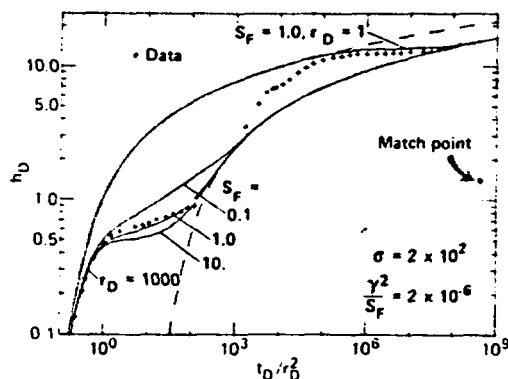


Fig. 12. Comparison of drawdown data to line source type curves using the transient block-to-fissure flow model with fracture skin and the indicated parameters.

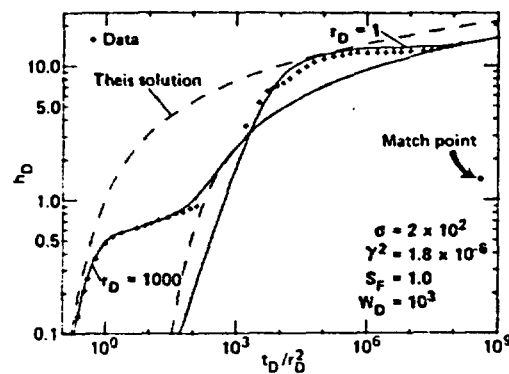


Fig. 13. Final type curve match using the transient block-to-fissure flow model with fracture skin and well bore storage.

appears justifiable in view of the following facts: (1) the major producing zone in the pumped well contains no casing, (2) the well was drilled using air and detergent as a circulation medium rather than drilling mud, and (3) the well was thoroughly developed by prior production testing.

Table 3 lists the values of the match point and dimensionless parameters obtained by the type curve match in Figure 13. The reservoir hydraulic properties are calculated from these values and are listed in Table 4. In calculating the fissure system hydraulic conductivity, the reservoir thickness, estimated from the bore hole flow survey, was taken to be 400 m. Also, as there were five major zones of water entry, the block thickness $2b'$ was estimated to be 80 m.

The fissure system hydraulic conductivity shown in Table 4 is consistent with values obtained by packer injection tests in the pumped well (Lahoud et al.) and with values obtained for fractured rock reservoirs in other locales [see Freeze and Cherry, 1979; Winograd and Thordarson, 1975]. Of the calculated parameters shown in Table 4, K and S , are probably the most reliable, given that the reservoir thickness was accurately estimated. Hydraulic conductivity obtained for the blocks is subject to uncertainty, largely because of the ratio r_w/b' , which is squared in the calculation. However, it is not inconsistent with values obtained by packer injection tests, given by Lahoud et al., that range from 3×10^{-8} to 6×10^{-6} m/s in intervals containing no major producing fractures. Laboratory measurements of the hydraulic conductivity of cores taken from the wells suggest that matrix hydraulic conductivity is two to five orders of magnitude less than that shown in Table 4 (Lahoud et al.). The value of block hydraulic conductivity obtained by this analysis, therefore, most likely represents the hydraulic conductivity of a system of joint surfaces within the matrix blocks. This suggestion is also supported by noting that the block specific storage shown in Table 4 appears to be two or three orders of magnitude larger than values calculated for unfractured rock [see van der Kamp and Gale, 1983, Table 1]. It can probably only be accounted for by the presence of highly compressible microfissures within the blocks. It should be pointed out that use of sphere-shaped blocks rather than slab-shaped blocks would reduce this discrepancy in block hydraulic conductivity by one order of magnitude but would not necessarily mean that sphere-shaped blocks are a more realistic descriptive model. Because it is not possible to determine the average fracture skin thickness, only the ratio K_s/b_s is given in Table 4.

The value of dimensionless well bore storage shown in Table 3 that was obtained by the type curve match in Figure

not even certain that blocks exist.

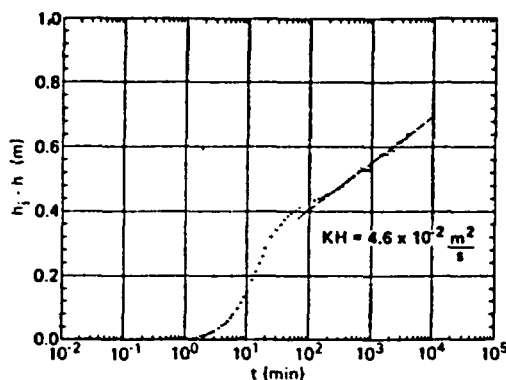


Fig. 14a. Evaluation of KH from a semilogarithmic plot of drawdown data for the observation well.

13 agrees closely with the calculated value of W_D . This provides an internal check on the calculated value of S_s or the estimated value of reservoir thickness H . Because effects of well bore storage are due to changing liquid level, and as in this test $r_c = r_w$, (21) reduces to

$$W_D = 1/2S_s H \quad (65)$$

The calculated value of W_D obtained from (65) is 833.

It was pointed out in the previous section that use of the semilogarithmic straight line method for evaluating KH in fractured rock reservoirs should be done with caution. As shown in Figure 14a, application of this approach to the observation well data in this field test produces a nice straight line and results in a value of fissure system hydraulic conductivity that is about an order of magnitude greater than that shown in Table 4. This is consistent with the present analysis if these data do indeed occur during the period of transition from early to late time under the influence of fracture skin. As shown in Figure 14b, a semilogarithmic plot of late time (greater than 1000 minutes), pumped well data also produces a straight line and results in a fissure system hydraulic conductivity that agrees (within 20%) with that shown in Table 4, as it should.

The results of this analysis support the contention that the flow system can be treated successfully with a double-porosity model. By incorporating fracture skin in the model, it appears possible to obtain an improved description of the flow system. Not included in the analysis are effects of partial penetration and anisotropy. The former is probably not important in this well test because the major zone of production shown in

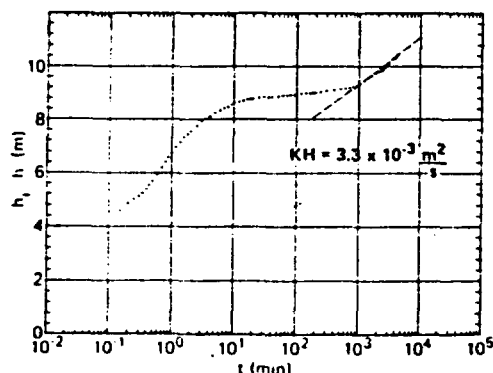


Fig. 14b. Evaluation of KH from a semilogarithmic plot of drawdown data for the pumped well.

Figure 9 appears to have been fully penetrated by the pumped well. Also, there is evidence that there is good hydraulic connection between producing zones. It is probable, however, that effects of anisotropy are significant. This is evident from field and bore hole observation of preferred fracture orientation and from the natural tendency of bore holes in the area to deviate from the vertical toward the west and southwest [Scott *et al.*, 1983]. A well test with data from a pumped well and a single observation well is insufficient to evaluate the hydraulic conductivity tensor. It is also possible that hydraulic boundaries due to major faults or intrusive dikes and sills are present within the flow regime. The change in slope that occurs at $t = 1000$ min in Figure 14b might possibly be interpreted by taking these factors into account; however, the change in slope is on the order of 10:1 rather than 2:1, which is characteristic of a single hydraulic boundary. Also, as the data appear to be consistent with the assumptions of the proposed double-porosity model, it is not necessary to call upon added complications.

CONCLUSIONS

The prevailing theories of flow to a well in a double-porosity reservoir can be unified by accounting for fracture skin in the mathematical development. When modified to include fracture skin, the commonly used theory based on the assumption of pseudo-steady state block-to-fissure flow is found to be a special case of the theory presented in this paper that is based on the assumption of transient block-to-fissure flow with fracture skin. The former applies when fracture skin hydraulic conductivity is sufficiently low. In the absence of fracture skin the pseudo-steady state flow model does not have a sound theoretical basis.

The interference test data from the Nevada Test Site support the hypothesis that fracture skin may be important in some double-porosity systems. Use of the modified theory for analysis of these data shows the advantage of applying the transient block-to-fissure flow assumption: it is possible to evaluate "block" hydraulic conductivity if the fracture skin hydraulic conductivity is not so low as to obscure the transient character of the flow in the blocks. Comparison of the result obtained by this analysis with laboratory measurements of block hydraulic conductivity suggests that the well test data analysis gives rise to an apparent block hydraulic conductivity that includes effects of microfractures within the matrix rock.

Caution is urged in the use of the standard semilogarithmic straight line method for evaluating the product of hydraulic conductivity and reservoir thickness in double-porosity systems. Analysis of the data in the report shows that the hydraulic conductivity of the fissure system may be overestimated by one order of magnitude if the straight line method is applied to the observation well data.

Because the presence or absence of fracture skin can have marked influence on the exchange of fluids between blocks and fissures, careful consideration of whether or not it is present should be given. This is especially true in regions under study for the disposal of toxic wastes. The presence of fracture skin due to mineral deposition or alteration will not only affect the adsorptive characteristics of the rocks but will also alter the flow velocities in the fractures and, consequently, the dispersive characteristics of the fractured rock system.

APPENDIX A

In this appendix the derivation of the Laplace transform solutions is given for the case of transient flow from slab-

shaped blocks with fracture skin. The coupled boundary value problems given by (24), (28), (32), (33), (35), and (36) for the fissure system and by (38)–(41) for the block system is put in nondimensional form by substituting the definitions for dimensionless drawdown, time and distance given by (12)–(14), (21), and (43)–(45) and the dimensionless parameters σ , γ , S_F , and m .

For the fissure system the governing differential equation becomes

$$\frac{\partial^2 h_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial h_D}{\partial r_D} = \frac{\partial h_D}{\partial t_D} + q_D' \quad r_D \geq 1 \quad (A1)$$

where

$$q_D' = \gamma^2 \left(\frac{\partial h_D'}{\partial z_D} \right)_{z_D=1} \quad (A2)$$

The initial condition is

$$h_D = 0 \quad r_D \geq 1 \quad (A3)$$

The boundary conditions are

$$h_D = 0 \quad r_D = \infty \quad (A4)$$

and

$$W_D \frac{\partial h_{wD}}{\partial t_D} - \frac{\partial h_D}{\partial r_D} = 2 \quad r_D = 1 \quad (A5)$$

where

$$h_{wD} = h_D - S_w \frac{\partial h_D}{\partial r_D} \quad r_D = 1 \quad (A6)$$

For the block system the governing differential equation becomes

$$\frac{\partial^2 h_D'}{\partial z_D^2} = \frac{\sigma}{\gamma^2} \frac{\partial h_D'}{\partial t_D} \quad 0 \leq z_D \leq 1 \quad (A7)$$

The initial condition is

$$h_D' = 0 \quad 0 \leq z_D \leq 1 \quad (A8)$$

The boundary conditions are

$$\partial h_D' / \partial z_D = 0 \quad z_D = 0 \quad (A9)$$

$$h_D' = h_D - S_F \frac{\partial h_D'}{\partial z_D} \quad z_D = 1 \quad (A10)$$

By performing the Laplace transformation procedure on (A1)–(A10), the coupled subsidiary boundary value problem is obtained. The subsidiary differential equation for the fissure system is

$$\frac{\partial^2 \bar{h}_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial \bar{h}_D}{\partial r_D} = p \bar{h}_D + \bar{q}_D' \quad r_D \geq 1 \quad (A11)$$

where

$$\bar{q}_D' = \gamma^2 \left(\frac{\partial \bar{h}_D'}{\partial z_D} \right)_{z_D=1} \quad (A12)$$

The subsidiary boundary conditions are

$$\bar{h}_D = 0 \quad r_D = \infty \quad (A13)$$

$$p W_D \bar{h}_{wD} - (\partial \bar{h}_D / \partial r_D) = 2/p \quad r_D = 1 \quad (A14)$$

where

$$\bar{h}_{wD} = \bar{h}_D - S_w \frac{\partial \bar{h}_D}{\partial r_D} \quad r_D = 1 \quad (A15)$$

The subsidiary differential equation for the block system is

$$\partial^2 \bar{h}_D' / \partial z_D^2 = m^2 \bar{h}_D' \quad 0 \leq z_D \leq 1 \quad (A16)$$

The subsidiary boundary conditions are

$$\partial \bar{h}_D' / \partial z_D = 0 \quad z_D = 0 \quad (A17)$$

$$\bar{h}_D' = \bar{h}_D - S_F \frac{\partial \bar{h}_D'}{\partial z_D} \quad z_D = 1 \quad (A18)$$

The general solution to (A11) is

$$\bar{h}_D' = A \cosh(z_D m) + B \sinh(z_D m) \quad (A19)$$

Applying the boundary conditions (A17) and (A18) the particular solution is

$$\bar{h}_D' = \frac{\bar{h}_D}{[1 + S_F m \tanh(m)]} \frac{\cosh(z_D m)}{\cosh(m)} \quad (A20)$$

Substitution of (A20) into (A11) and (A12), letting $\bar{q}_D' = \bar{q}_D \bar{h}_D$, yields

$$\frac{\partial^2 \bar{h}_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial \bar{h}_D}{\partial r_D} = p \bar{h}_D + \bar{q}_D \bar{h}_D \quad (A21)$$

where

$$\bar{q}_D = \frac{\gamma^2 m \tanh(m)}{1 + S_F m \tanh(m)} \quad (A22)$$

(A20) and (A22) are the solutions (47) and (46).

The general solution to (A21) is

$$\bar{h}_D = C I_0(r_D x) + D K_0(r_D x) \quad (A23)$$

where

$$x = (p + \bar{q}_D)^{1/2}$$

Since by (A13), \bar{h}_D is bounded, $C = 0$, and

$$\bar{h}_D = D K_0(r_D x) \quad (A24)$$

Applying (A14)

$$D = \frac{2/p - p W_D \bar{h}_{wD}}{x K_1(x)} \quad (A25)$$

By applying (A15) and by algebraic manipulation, the particular solutions are

$$\bar{h}_{wD} = \frac{2[K_0(x) + x S_w K_1(x)]}{p\{p W_D [K_0(x) + x S_w K_1(x)] + x K_1(x)\}} \quad (A26)$$

$$\bar{h}_D = \frac{2 K_0(r_D x)}{p\{p W_D [K_0(x) + x S_w K_1(x)] + x K_1(x)\}} \quad (A27)$$

(A26) and (A27) are the solutions (19) and (20), where

$$x = (p + \bar{q}_D)^{1/2}$$

and \bar{q}_D is defined by (A22).

APPENDIX B

In this appendix the derivation of the Laplace transform solutions is given for the case of transient flow from sphere-shaped blocks with fracture skin. The coupled boundary value problems given by (24), (31)–(33), (35), and (36) for the fissure system and by (48)–(51) for the block system are put in nondimensional form, as in Appendix A. Dimensionless distance in the block is defined by (52).

The boundary value problem for the fissure system is the same as (A1)–(A6) with the exception that (A2) is replaced by

$$q_D' = 3\gamma'^2 \left(\frac{\partial h_D'}{\partial \rho_D} \right)_{\rho_D=1} \quad (\text{B1})$$

For the block system the governing differential equation becomes

$$\frac{\partial^2 (\rho_D h_D')}{\partial \rho_D^2} = \frac{\sigma}{\gamma'^2} \frac{\partial (\rho_D h_D')}{\partial t} \quad 0 \leq \rho_D \leq 1 \quad (\text{B2})$$

The initial condition is

$$h_D' = 0 \quad 0 \leq \rho_D \leq 1 \quad (\text{B3})$$

The boundary conditions are

$$h_D' \text{ finite} \quad \rho_D = 0 \quad (\text{B4})$$

$$h_D' = h_D - S_F \frac{\partial h_D'}{\partial \rho_D} \quad \rho_D = 1 \quad (\text{B5})$$

The subsidiary boundary value problem for the fissure system is the same as (A11)–(A15) with the exception that (A12) is replaced by

$$\bar{q}_D' = 3\gamma'^2 \left(\frac{\partial \bar{h}_D'}{\partial \rho_D} \right)_{\rho_D=1} \quad (\text{B6})$$

The subsidiary differential equation for the block system is

$$\frac{d^2 (\rho_D \bar{h}_D')}{d\rho_D^2} = m^2 (\rho_D \bar{h}_D') \quad 0 \leq \rho_D \leq 1 \quad (\text{B7})$$

The subsidiary boundary conditions are

$$\bar{h}_D' \text{ finite} \quad \rho_D = 0 \quad (\text{B8})$$

$$\bar{h}_D' = \bar{h}_D - S_F \frac{d\bar{h}_D'}{d\rho_D} \quad \rho_D = 1 \quad (\text{B9})$$

The general solution to (B7) is

$$\rho_D \bar{h}_D' = A \cosh(\rho_D m) + B \sinh(\rho_D m) \quad (\text{B10})$$

For (B8) to hold, $A = 0$ and

$$\bar{h}_D' = \frac{B}{\rho_D} \sinh(\rho_D m) \quad (\text{B11})$$

By applying boundary condition (B9), the particular solution is

$$\bar{h}_D' = \frac{\bar{h}_D}{\rho_D \{1 + S_F [m \coth(m) - 1]\}} \frac{\sinh(\rho_D m)}{\sinh(m)} \quad (\text{B12})$$

The subsidiary differential equation for the fissure system is (A21). Again, as in Appendix A, by letting $\bar{q}_D' = \bar{q}_D \bar{h}_D$,

$$\bar{q}_D = \frac{3\gamma'^2 [m \coth(m) - 1]}{1 + S_F [m \coth(m) - 1]} \quad (\text{B13})$$

(B12) and (B13) are the solutions (53) and (54). The Laplace transform solutions for the dimensionless drawdown in the fissure system are given by (A26) and (A27) in Appendix A with \bar{q}_D as defined in (B13).

APPENDIX C

In this appendix derivations are given of the Laplace transform solutions for the case of pseudo-steady state flow from slab-shaped and sphere-shaped blocks with fracture skin. By neglecting the left-hand side of (6) and combining the result

with (5), the controlling differential equation becomes, in radial coordinates,

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S_s}{K} \frac{\partial h}{\partial t} + \frac{S_s'}{K} \frac{\partial h'}{\partial t} \quad (\text{C1})$$

An additional equation is obtained by equating (55) and (56) so that

$$\frac{S_s'}{K} \frac{\partial h'}{\partial t} = \frac{K_s V_f}{K b b_s} (h_D - h_D') \quad (\text{C2})$$

After substituting the various dimensionless parameters, (C1) and (C2) become

$$\frac{\partial^2 h_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial h_D}{\partial r_D} = \frac{\partial h_D}{\partial t_D} + \frac{\sigma \partial h_D'}{\partial t_D} \quad (\text{C3})$$

$$\sigma \frac{\partial h_D'}{\partial t_D} = \lambda (h_D - h_D') \quad (\text{C4})$$

where

$$\lambda = \frac{\gamma'^2 V_f}{S_F} \frac{b'}{b} \quad (\text{C5})$$

Performing the Laplace transformation procedure on (C3) and (C4) and combining the results the subsidiary differential equation becomes

$$\frac{\partial^2 \bar{h}_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial \bar{h}_D}{\partial r_D} = \rho \bar{h}_D + q_D \bar{h}_D \quad (\text{C6})$$

where

$$\bar{q}_D = \frac{\sigma \rho \lambda}{\sigma \rho + \lambda} \quad (\text{C7})$$

For the slab-shaped blocks, (26) applies, and (C7) becomes

$$\bar{q}_D = \frac{\gamma'^2 m^2}{1 + S_F m^2} \quad (\text{C8})$$

For sphere-shaped blocks, (29) applies, and (C7) becomes

$$\bar{q}_D = \frac{3\gamma'^2 m^2}{3 + S_F m^2} \quad (\text{C9})$$

NOTATION

- 2b average aperture of fissures, L .
- 2b' average thickness or diameter of blocks, L .
- b_s average thickness of fracture skin, L .
- C well bore storage coefficient, L^2 .
- C_{obs} observed compressibility of fluid well system, LT^2M^{-1} .
- g acceleration of gravity, LT^{-2} .
- H reservoir thickness, L .
- h hydraulic head in fissure, L .
- h' hydraulic head in block, L .
- h_i initial hydraulic head, L .
- h_w hydraulic head in pumped well, L .
- I_0 modified Bessel function of the first kind and order zero.
- K_0 modified Bessel function of the second kind and order zero.
- K_1 modified Bessel function of the second kind and order unity.
- K hydraulic conductivity of the fissure system, LT^{-1} .
- K' hydraulic conductivity of the block system, LT^{-1} .
- K_f hydraulic conductivity of an average fissure, LT^{-1} .
- K_m hydraulic conductivity of an average block, LT^{-1} .

- K_f hydraulic conductivity of fracture skin, LT^{-1} .
 n number of producing fissures.
 p Laplace transform variable.
 Q_f discharge to the pumped well from an average fissure, L^3T^{-1} .
 Q_T total well discharge, L^3T^{-1} .
 q source term for transient block-to-fissure flow, T^{-1} .
 q_s source term for pseudo-steady state block-to-fissure flow, T^{-1} .
 r radial distance measured from center of pumped well, L .
 r_w effective radius of pumped well, L .
 r_c internal radius of pumped well casing, L .
 S_f specific storage of the fissure system, L^{-1} .
 S_b specific storage of the block system, L^{-1} .
 S_{f1} specific storage of an average fissure, L^{-1} .
 S_{bm} specific storage of an average block, L^{-1} .
 S_w dimensionless well bore skin.
 S_F dimensionless fracture skin.
 t time since start of pumping, T .
 V_f ratio of total volume of fissures to the bulk volume.
 V_m ratio of total volume of blocks to the bulk volume.
 V_w volume of liquid in a pressurized well, L^3 .
 W_D dimensionless well bore storage.
 y generalized block coordinate, L .
 z distance measured from center of a slab-shaped block to the fissure, L .
 α relates to geometry of blocks under assumption of pseudo-steady state flow, L^{-2} .
 ρ_w density of fluid in the pumped well, ML^{-3} .
 ρ distance measured from center of a sphere-shaped block to the fissure, L .
 D (as a subscript) dimensionless parameter.

Dimensionless Groupings for Reservoir Parameters

$$\gamma = (r_w/b')(K'/K)^{1/2}$$

$$m = (\sigma p)^{1/2}/\gamma$$

$$\sigma = S_f/S_b$$

$$S_F = K'b_s/K_b b'$$

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